Explainability Index (EI) & Risk of Target (RoT)

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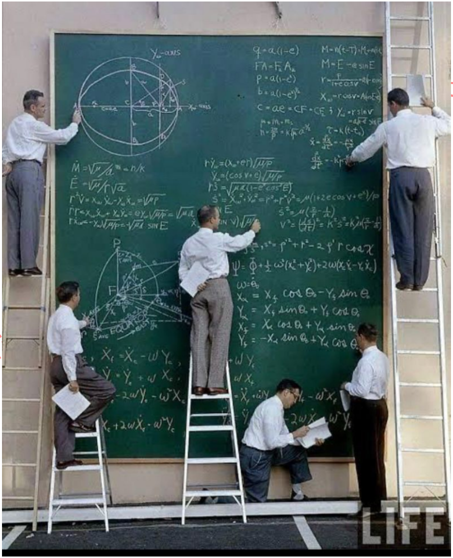
Picture of the Day



Quote of the Day

Successful investing is about managing risk, not avoiding it.

`Benjamin Graham`, father of value investing

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Introduction (1 of 2)

- 100s of proposed performance measures (ranging from very simple to more advanced, measured in different scales, linear/non-linear, etc) are used to assess securities, evaluate portfolios, create asset allocation profiles, capital adequacy/efficiency, risk management and so on (e.g., return, `VaR`, `Sharpe`, `Calmar`, etc)
- assessments hinge on the relative range of individual performance measures, and usage is based on some form of a grid of select measures with associated weights

Introduction (2 of 2)

- we propose a *Unifying Framework of Performance Measures* as an `Explainability Index (EI)` that captures the multi-dimensionality and nuances measured by the individual measures, where it balances the different input categories of performance measures according to default or specified preferences and gives a composite bounded score between 0 and 1.

- we also propose a relative measure as the `Risk of Target (RoT)` that leverages the `EI` for comparing the performance of assets/portfolios/etc with their targets and assesses the drivers of divergence.

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Scaling

- first realization is that performance measures do not have the same scale
  - ▷ making their direct comparison non-practical
- 1st step: scaling of performance measures
  - ▷ use a `sigmoid` function to do so (scale values between 0 & 1)

Scaling - Sigmoid

- a separate issue rises with the sigmoid function; given the nature of the value of the features, some performance measures get mapped into bucketed regions
- e.g. `Batting Average` has a range of values between 0% and 100%, resulting in a value between 0.5 and 1 after applying a sigmoid function[1]
- on the other hand, `Max Drawdown` has a range between -100% and 0%, resulting on a value between 0 and 0.5
- therefore, cannot compare scaled values of these performance measures

[1] for volatility would result in a value greater than 0.5

Two-step transformation (1 of 2)

- for the sigmoid transformation to properly scale the measures, we apply a linear transformation beforehand
- linear mapping that aligns $\alpha \times 100\%$ & $(1-\alpha) \times 100\%$ of sigmoid with corresponding historical distribution respectively
- required values from the sigmoid function are obtained by:

$$\frac{1}{1+e^{-x_\alpha}} = \alpha$$

solving for x_α to get

$$x_\alpha = -\ln\left(\frac{1-\alpha}{\alpha}\right)$$

and

$$x_{1-\alpha} = \ln\left(\frac{1-\alpha}{\alpha}\right)$$

Two-step transformation (2 of 2)

- the linear transformation is given by

$$\hat{m} = \beta(m - m_\alpha) - x_\alpha$$

where

$$\beta = \frac{x_{1-\alpha} - x_\alpha}{m_{1-\alpha} - m_\alpha}$$

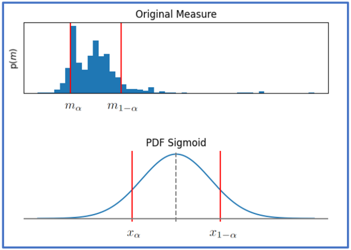
and m_α and $m_{1-\alpha}$ are from historical distribution

- for this step, values could come from the asset, the index, or a pool of multiple assets/indexes

- thus we obtain

$$\tilde{m} = \frac{1}{1 + e^{-\hat{m}}}$$

Transformation Example



I

Linear Transformation

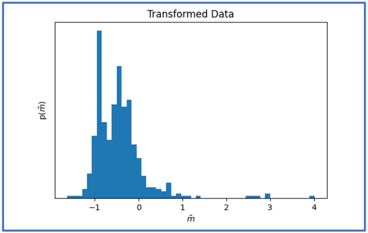
$$\beta = \frac{x_{1-\alpha} - x_\alpha}{m_{1-\alpha} - m_\alpha}$$

Where β is slope, mapping:

$$\hat{m} - x_\alpha = \beta(m - m_\alpha)$$

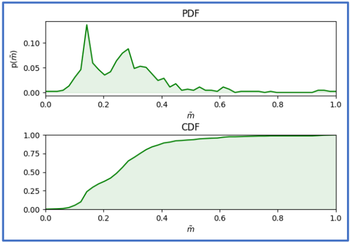
With m as original, and $\hat{m}$ as transformed via linear mapping

II



III

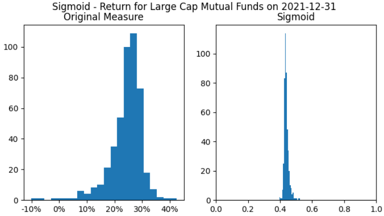
$\tilde{m} = \frac{1}{1+e^{-\tilde{m}}}$

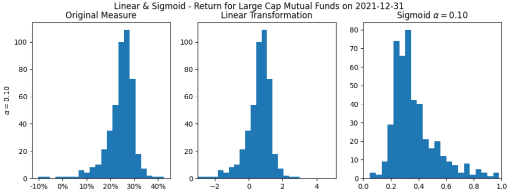


Proper Orientation of Performance Measures

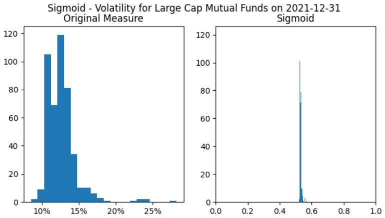
- when building an index, we need to take into account the importance of the performance orientation
- for performance measures like `Volatility` the lower the better and for `Return` the higher the better
- by construction, `EI` assumes 0 is better than 1, so direction needs to be adjusted accordingly
- in case of the higher the better, define $\bar{m} = 1 - \tilde{m}$, otherwise $\bar{m} = \tilde{m}$
- after this adjustment, all performance measures have the same scale and orientation

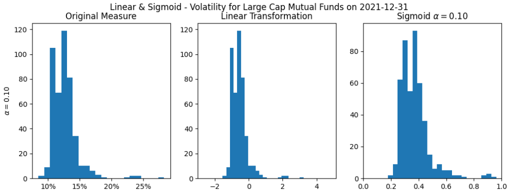
Transformation Example - Return



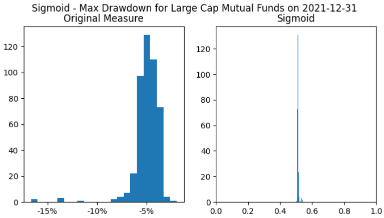


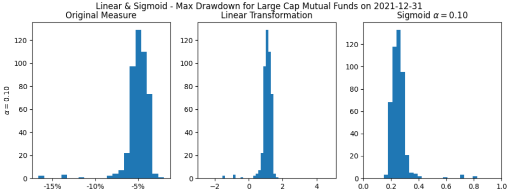
Transformation Example - Volatility



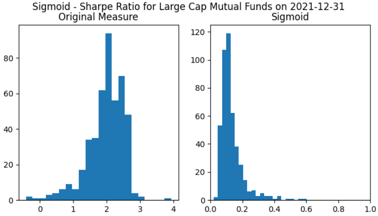


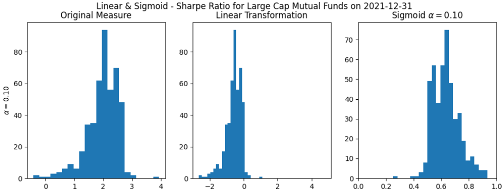
Transformation Example - Max Drawdown





Transformation Example - Sharpe Ratio





Categories

- in order to enhance explainability of `EI`, we define categories based on user preferences
- four default categories are: return, volatility, drawdown, and alternatives
- within each category, all transformed performance measures are equally weighted which yields a number for each category
- when it comes to combining categories, users can use their own weights based on their preferences (default is equally weighted)

Explainability Index (EI)

- to unify all categories into a single value, i.e. EI, users can employ one of the below methods:
- arithmetic: $\sum_{k=1}^{K} w_k \bar{m}_k$
- geometric: $\prod_{k=1}^{K} \bar{m}_k^{w_k}$
- distributional: $1 - (1 - \mathtt{EI}) \times (1 - d_H)$
  - d_H is Hellinger distance
  - $H^2(\mu_1, \mu_2, \sigma_1, \sigma_2) = 1 - \sqrt{\frac{2\sigma_1\sigma_2}{\sigma_1^2+\sigma_2^2}} e^{-\frac{1}{4}\frac{(\mu_1-\mu_2)^2}{\sigma_1^2+\sigma_2^2}}$
  - $d_H = \frac{1}{N}\sum_{i=1}^{N} H^2\left(R^{(i)}, R\right)$

Risk of Target (RoT)

- comparing EI of an asset with EI of its benchmark would allow portfolio managers to assess their performance deviation from the target benchmark i.e. RoT
- we can calculate RoT as a difference

$$\mathrm{RoT} = \mathrm{EI}^{asset} - \mathrm{EI}^{target}$$

or as a percentage difference

$$\mathrm{RoT} = \frac{\mathrm{EI}^{asset} - \mathrm{EI}^{target}}{\mathrm{EI}^{target}}$$

User-defined Parameters

- here are user-defined parameters/preferences
  - type of performance measures
  - level of α for sigmoid transformation
  - asset or group of assets used to determine z_α and $z_{1-\alpha}$
  - categories
  - weights
  - EI methodology
  - variation across time[2]

[2] 2nd degree vs. 1st degree

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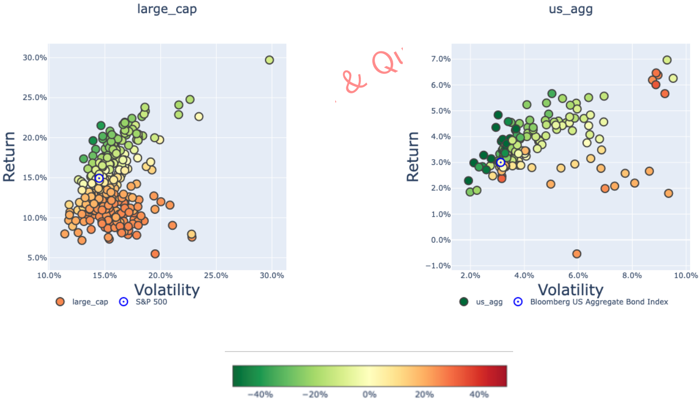
Example 1 – Efficient Frontier

- efficient frontier compares assets in 2D, return & volatility
- this could create undesirable risks

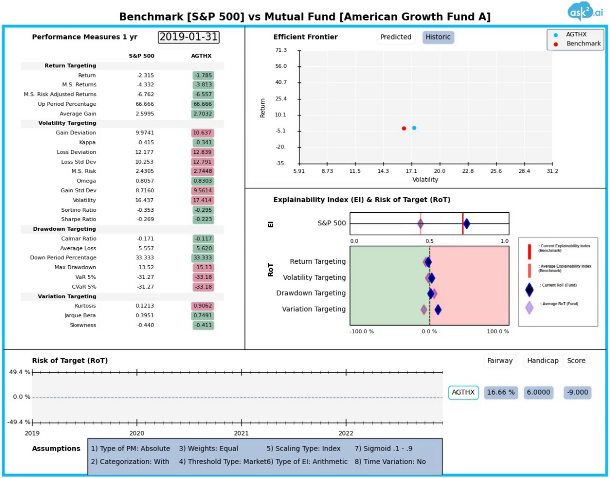
RoT Efficient Frontier

- RoT adds color for better visualization purposes
- it improves understanding in higher dimensional hidden risk



RoT Fund Analysis

this is a deeper dive into one of the funds (can be for any asset)



	S&P 500	AGTHX
Return Targeting		
Return	-2.315	-1.785
M.S. Returns	-4.332	-3.813
M.S. Risk Adjusted Returns	-6.762	-6.557
Up Period Percentage	66.666	66.666
Average Gain	2.5995	2.7032
Volatility Targeting		
Gain Deviation	9.9741	10.637
Kappa	-0.415	-0.341
Loss Deviation	12.177	12.839
Loss Std Dev	10.253	12.791
M.S. Risk	2.4305	2.7448
Omega	0.8057	0.8303
Gain Std Dev	8.7160	9.5614
Volatility	16.437	17.414
Sortino Ratio	-0.353	-0.295
Sharpe Ratio	-0.269	-0.223
Drawdown Targeting		
Calmar Ratio	-0.171	-0.117
Average Loss	-5.557	-5.620
Down Period Percentage	33.333	33.333
Max Drawdown	-13.52	-15.13
VaR 5%	-31.27	-33.18
CVaR 5%	-31.27	-33.18
Variation Targeting		
Kurtosis	0.1213	0.9062
Jarque Bera	0.3951	0.7491
Skewness	-0.440	-0.411

RoT Multi Fund Analysis: Index Tracking ETFs (1 of 2)

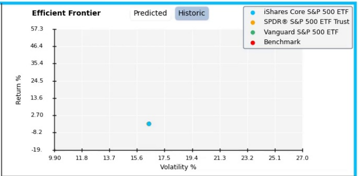
- it makes RoT an ideal tool to compare how accurately an ETF can replicate the behavior of an index
- index tracking ETFs should have a nearly identical holding composition to their respective index but affected by fees, taxes, and transaction cost
- given that there are multiple index tracking ETFs, it is crucial to assess which one follows closely their respective index

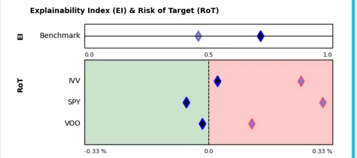
RoT Multi Fund Analysis: Index Tracking ETFs (2 of 2)

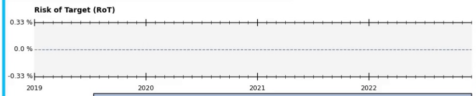
Benchmark [S&P 500] vs ETFs

Performance Measures 1 yr — 2019-01-31

	Benchmark	IVV	SPY	VOO
Return Targeting				
Return	-2.315	-2.358	-2.385	-2.352
M.S. Returns	-4.332	-4.373	-4.400	-4.368
M.S. Risk Adj Returns	-6.762	-6.802	-6.813	-6.796
Up Period Percentage	66.666	66.666	66.666	66.666
Average Gain	2.5995	2.5957	2.5850	2.5964
Volatility Targeting				
Gain Deviation	9.9741	9.9648	9.9387	9.9685
Kappa	-0.415	-0.418	-0.423	-0.418
Loss Deviation	12.177	12.183	12.144	12.177
Loss Std Dev	10.253	10.250	10.174	10.220
M.S. Risk	2.4305	2.4290	2.4127	2.4280
Omega	0.8057	0.8040	0.8023	0.8042
Gain Std Dev	8.7160	8.7134	8.7065	8.7176
Volatility	16.437	16.435	16.386	16.433
Sortino Ratio	-0.353	-0.356	-0.360	-0.356
Sharpe Ratio	-0.269	-0.271	-0.274	-0.271
Drawdown Targeting				
Calmar Ratio	-0.171	-0.174	-0.176	-0.174
Average Loss	-5.557	-5.561	-5.548	-5.561
Down Period Percentage	33.333	33.333	33.333	33.333
Max Drawdown	-13.52	-13.52	-13.48	-13.51
VaR 5%	-31.27	-31.28	-31.10	-31.24
Variation Targeting				
Kurtosis	0.1213	0.1204	0.1110	0.1160
Jarque Bera	0.3951	0.3955	0.3818	0.3900
Skewness	-0.440	-0.440	-0.433	-0.437







	Fairway	Handicap	Score
IVV	100.0 %	12.000	12.000
SPY	100.0 %	10.000	10.000
VOO	100.0 %	11.500	10.000

Assumptions

1) Type of PM: Absolute	3) Weights: Equal	5) Scaling Type: Index	7) Sigmoid .1 - .9
2) Categorization: With	4) Threshold Type: Market	6) Type of EI: Arithmetic	8) Time Variation: No

Example 2 – RoT of Portfolios (1 of 2)

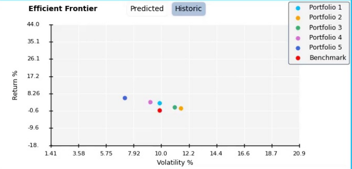
- RoT is ideal to assess portfolios against target objectives
- it is possible to combine RoT values of individual assets given that it is highly non-linear
- a portfolio RoT requires the calculation of all measures from portfolio monthly returns
- when presented with multiple portfolios that are in proximity to each other in the efficient frontier, RoT can portray their entire risk profile

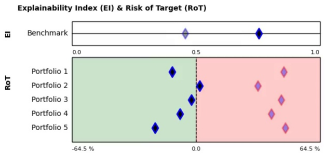
Example 2 – RoT of Portfolios (2 of 2)

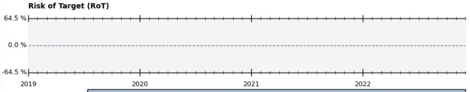
Benchmark [60% S&P 500 40% US Agg] vs Portfolios ask².ai

Performance Measures 1 yr 2019-01-31

	Benchmark	Port 1	Port 2	Port 3	Port 4	Port 5
Return Targeting						
Return	-0.488	3.3339	0.6868	1.1559	4.0192	6.1563
M.S. Returns	-2.543	1.2006	-1.391	-0.932	1.8718	3.9647
M.S. Risk Adj Returns	-3.434	0.3108	-2.605	-2.036	1.0966	3.4766
Up Period Percentage	58.333	66.666	58.333	66.666	58.333	66.666
Average Gain	1.8546	1.7412	2.1749	1.8419	2.1137	1.6852
Volatility Targeting						
Gain Deviation	6.3601	7.5767	7.8539	7.7601	6.9794	5.9586
Kappa	-0.416	0.2322	-0.204	-0.148	0.3980	1.1388
Loss Deviation	7.0965	5.9005	7.8954	7.3123	5.5255	3.9387
Loss Std Dev	6.9433	5.6654	8.2763	6.3441	4.9868	2.9184
M.S. Risk	0.8911	0.8898	1.2141	1.1039	0.7751	0.4880
Omega	0.8147	1.1107	0.9047	0.9314	1.1674	1.4950
Gain Std Dev	5.6717	7.4762	7.4817	7.4644	5.8460	4.6431
Volatility	9.9533	9.9669	11.625	11.124	9.2055	7.2202
Sortino Ratio	-0.349	0.1971	-0.172	-0.124	0.3257	0.9493
Sharpe Ratio	-0.260	0.1230	-0.122	-0.085	0.2076	0.5606
Drawdown Targeting						
Calmar Ratio	-0.063	0.5834	0.0779	0.1479	0.8951	2.1503
Average Loss	-2.635	-2.598	-2.834	-3.307	-2.117	-1.831
Down Period Percentage	41.666	33.333	41.666	33.333	41.666	33.333
Max Drawdown	-7.636	-5.714	-8.810	-7.814	-4.489	-2.862
VaR 5%	-16.22	-14.87	-19.07	-17.29	-11.74	-9.917
Variation Targeting						
Kurtosis	-0.234	2.2065	0.7026	1.0832	0.1521	-0.432
Jarque Bera	0.1483	3.3227	0.2499	0.7495	0.1273	0.0950
Skewness	-0.245	0.6664	0.0392	0.2852	0.2405	0.0271







	Fairway	Handicap	Score
Port 1	16.66 %	23.500	15.000
Port 2	75.0 %	18.750	14.000
Port 3	83.33 %	21.250	6.0000
Port 4	16.66 %	18.500	16.000
Port 5	8.333 %	13.250	14.000

Assumptions

1) Type of PM: Absolute	3) Weights: Equal	5) Scaling Type: Index	7) Sigmoid .1 - .9
2) Categorization: With	4) Threshold Type: Market	6) Type of EI: Arithmetic	8) Time Variation: No

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Conclusion - power of EI & RoT

- unify processes, assessments, and explanations
- add color and capture nuances (linear/non-linear) in a simple and explainable manner
- compare assets/portfolios in a uniform manner at a point in time (relative or trend)
- construct multi-objective asset allocation profiles/portfolios
- extend to incorporate any/all performance measures, alternative measures (e.g., `Expense Ratios`), holdings, etc